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Ex. 4.4

1) (i) $\dfrac{3}{4\sqrt{3}}$

$= \dfrac{3}{4\sqrt{3}} \times \dfrac{\sqrt{3}}{\sqrt{3}}$

$= \dfrac{3\sqrt{3}}{4(\cancel{\sqrt{3}})^{\cancel{2}}}$

$= \dfrac{\cancel{3}\sqrt{3}}{4 \times \cancel{3}} \Rightarrow \dfrac{\sqrt{3}}{4}$

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ii) $\dfrac{14}{\sqrt{98}}$

$$= \frac{14}{\sqrt{98}} \times \frac{\sqrt{98}}{\sqrt{98}}$$

$$= \frac{14\sqrt{98}}{(\sqrt{98})^2}$$

$$= \frac{\cancel{14}^{\,1} \times \sqrt{98}}{\cancel{98}\,7} \Rightarrow \frac{\sqrt{98}}{7} \text{ Ans}$$

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iv) $\dfrac{1}{3+2\sqrt{5}}$

$\because (a+b)(a-b)=a^2-b^2$

$= \dfrac{1}{3+2\sqrt{5}} \times \dfrac{3-2\sqrt{5}}{3-2\sqrt{5}}$

$= \dfrac{3-2\sqrt{5}}{(3)^2-(2\sqrt{5})^2}$

$= \dfrac{3-2\sqrt{5}}{9-4\times 5}$

$= \dfrac{3-2\sqrt{5}}{9-20}$

$= \dfrac{3-2\sqrt{5}}{-11}$

Ans

iii) $\frac{6}{\sqrt{8}\cdot\sqrt{27}}$

$= \frac{6}{\sqrt{8}\cdot\sqrt{27}} \times \frac{\sqrt{8}\cdot\sqrt{27}}{\sqrt{8}\cdot\sqrt{27}}$

$= \frac{6\left(\sqrt{8}\cdot\sqrt{27}\right)}{\left(\sqrt{8}\cdot\sqrt{27}\right)^{2}}$

$= \frac{{}^{1}3\cancel{6}\left(\sqrt{216}\right)}{\cancel{216}}$

$\cancel{108}$

36

$\Rightarrow \frac{\sqrt{216}}{36}$ Ans

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v) $\dfrac{15}{\sqrt{31}-4}$

$$= \frac{15}{\sqrt{31}-4} \times \frac{\sqrt{31}+4}{\sqrt{31}+4}$$

$$= \frac{15(\sqrt{31}+4)}{(\sqrt{31})^2 - (4)^2}$$

$$= \frac{15(\sqrt{31}+4)}{31-16}$$

$$= \frac{\cancel{15}(\sqrt{31}+4)}{\cancel{15}}$$

$$= \sqrt{31}+4$$

Ans

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vi) $\dfrac{2}{\sqrt{5}-\sqrt{3}}$

$$= \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{2(\sqrt{5}+\sqrt{3})}{5-3}$$

$$= \frac{\cancel{2}(\sqrt{5}+\sqrt{3})}{\cancel{2}}$$

$$= \sqrt{5}+\sqrt{3}$$

vii) $\dfrac{\sqrt{3}-1}{\sqrt{3}+1}$

$$= \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$$

$$= \frac{(\sqrt{3})^2+(1)^2-2(\sqrt{3})(1)}{3-1}$$

$\because (a-b)^2 = a^2+b^2-2ab$

$$= \frac{3+1-2\sqrt{3}}{2}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{\cancel{2}(2-\sqrt{3})}{\cancel{2}}$$

$$= \underline{\underline{2-\sqrt{3}}}$$

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viii) $\dfrac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(\sqrt{5}+\sqrt{3})^2}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{3})^2+2(\sqrt{5})(\sqrt{3})}{5-3}$$

$$= \frac{5+3+2\sqrt{15}}{2}$$

$$= \frac{8+2\sqrt{15}}{2}$$

$$= \frac{\cancel{2}(4+2\sqrt{15})}{\cancel{2}}$$

$$= 4+2\sqrt{15} \text{ Ans}$$

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② Find the conjugate.

i) $3+\sqrt{7}$

Conjugate $= 3-\sqrt{7}$

ii) $4-\sqrt{5}$

Conjugate $= 4+\sqrt{5}$

iii) $2+\sqrt{3}$

Conjugate $= 2-\sqrt{3}$

iv) $2+\sqrt{5}$

conjugate $= 2-\sqrt{5}$

v) $5+\sqrt{7}$

conjugate $= 5-\sqrt{7}$

vi) $4-\sqrt{15}$

Conjugate $= 4+\sqrt{15}$

vii) $7-\sqrt{6}$

Conjugate $= 7+\sqrt{6}$

viii) $9+\sqrt{2}$

Conjugate $= 9-\sqrt{2}$

3) (i) $x = 2-\sqrt{3}$ $\quad \frac{1}{x} = ?$

Sol:-

$$x = 2-\sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$\frac{1}{x} = \frac{2+\sqrt{3}}{4-3}$$

$$\frac{1}{x} = 2+\sqrt{3}$$

Ans.

3) iii) $x = 4 - \sqrt{17}$ $\qquad \frac{1}{x} = ?$

$$\boxed{\frac{1}{x} = -4 - \sqrt{17}}$$

Sol:

$$x = 4 - \sqrt{17}$$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}}$$

$$\frac{1}{x} = \frac{1}{4-\sqrt{17}} \times \frac{4+\sqrt{17}}{4+\sqrt{17}}$$

$$\frac{1}{x} = \frac{4+\sqrt{17}}{(4)^2 - (\sqrt{17})^2}$$

$$\Rightarrow \frac{1}{x} = \frac{4+\sqrt{17}}{16-17}$$

$$\Rightarrow \frac{1}{x} = \frac{4+\sqrt{17}}{-1}$$

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3) iii) $x = \sqrt{3} + 2$ $\quad x + \frac{1}{x} = ?$

Sol:-

$$x = \sqrt{3} + 2$$

$$\frac{1}{x} = \frac{1}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$$

$$\frac{1}{x} = \frac{\sqrt{3}-2}{(\sqrt{3})^2 - (2)^2}$$

$$\frac{1}{x} = \frac{\sqrt{3}-2}{3-4}$$

$$\frac{1}{x} = \frac{\sqrt{3}-2}{-1}$$

$$\frac{1}{x} = 2 - \sqrt{3}$$

$$x + \frac{1}{x} = \cancel{\sqrt{3}} + 2 + 2 \cancel{- \sqrt{3}}$$

$$\boxed{x + \frac{1}{x} = 4}$$

4) (i) $\dfrac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} + \dfrac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}}$

$$= \frac{1+\sqrt{2}}{\sqrt{5}+\sqrt{3}} \times \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}} + \frac{1-\sqrt{2}}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$$

$$= \frac{(1+\sqrt{2})(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{(1-\sqrt{2})(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{5-3} + \frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{5-3}$$

$$= \frac{\sqrt{5}-\sqrt{3}+\sqrt{10}-\sqrt{6}}{2} + \frac{\sqrt{5}+\sqrt{3}-\sqrt{10}-\sqrt{6}}{2}$$

$$= \frac{\sqrt{5}\cancel{-\sqrt{3}}\cancel{+\sqrt{10}}\cancel{-\sqrt{6}}+\sqrt{5}\cancel{+\sqrt{3}}\cancel{-\sqrt{10}}\cancel{-\sqrt{6}}}{2}$$

$$= \frac{\cancel{2}\sqrt{5}}{\cancel{2}}$$

$$= \sqrt{5} \text{ Ans}$$

ii) $\dfrac{1}{2+\sqrt{3}} + \dfrac{2}{\sqrt{5}-\sqrt{3}} + \dfrac{1}{2+\sqrt{5}}$

$$= \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}} + \frac{2}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}} + \frac{1}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$$

$$= \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2} + \frac{2(\sqrt{5}+\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2} + \frac{2-\sqrt{5}}{(2)^2-(\sqrt{5})^2}$$

$$= \frac{2-\sqrt{3}}{4-3} + \frac{\cancel{2}(\sqrt{5}+\sqrt{3})}{\cancel{2}} + \frac{2-\sqrt{5}}{4-5}$$

$$= 2-\sqrt{3} + \sqrt{5}+\sqrt{3} + \frac{2-\sqrt{5}}{-1}$$

$$= \cancel{2} \cancel{-\sqrt{3}} + \sqrt{5} \cancel{+\sqrt{3}} \cancel{-2} + \sqrt{5}$$

$$= 2\sqrt{5} \quad \text{Ans.}$$

iii) $\dfrac{2}{\sqrt{5}+\sqrt{3}}+\dfrac{1}{\sqrt{3}+\sqrt{2}}-\dfrac{3}{\sqrt{5}+\sqrt{2}}$

$$=\frac{2}{\sqrt{5}+\sqrt{3}}\times\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}-\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{2}}\times\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}-\sqrt{2}}-\frac{3}{\sqrt{5}+\sqrt{2}}\times\frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$=\frac{2(\sqrt{5}-\sqrt{3})}{(\sqrt{5})^2-(\sqrt{3})^2}+\frac{\sqrt{3}-\sqrt{2}}{(\sqrt{3})^2-(\sqrt{2})^2}-\frac{3(\sqrt{5}-\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$=\frac{\cancel{2}(\sqrt{5}-\sqrt{3})}{\cancel{2}}+\frac{\sqrt{3}-\sqrt{2}}{1}-\frac{\cancel{3}(\sqrt{5}-\sqrt{2})}{\cancel{3}}$$

$$=\cancel{\sqrt{5}}\cancel{-\sqrt{3}}\cancel{+\sqrt{3}}\cancel{-\sqrt{2}}\cancel{-\sqrt{5}}\cancel{+\sqrt{2}}$$

$$=0 \text{ Ans}$$

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5) $x = 2+\sqrt{3}$ $\quad x - \frac{1}{x} = ?$ $\quad \left(x-\frac{1}{x}\right)^2 = ?$

$$x = 2+\sqrt{3}$$

$$\frac{1}{x} = \frac{1}{2+\sqrt{3}} \times \frac{2-\sqrt{3}}{2-\sqrt{3}}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{(2)^2-(\sqrt{3})^2}$$

$$\frac{1}{x} = \frac{2-\sqrt{3}}{4-3}$$

$$\frac{1}{x} = 2-\sqrt{3}$$

$$x - \frac{1}{x} = (2+\sqrt{3}) - (2-\sqrt{3})$$

$$= \cancel{2}+\sqrt{3} \cancel{-2}+\sqrt{3}$$

$$\boxed{x-\frac{1}{x} = 2\sqrt{3}}$$

$$\left(x-\frac{1}{x}\right)^2 = \left(2\sqrt{3}\right)^2$$

$$\left(x-\frac{1}{x}\right)^2 = 4\times 3$$

$$\boxed{\left(x-\frac{1}{x}\right)^2 = 12}$$

5) ii) $x = \dfrac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$ $\qquad x+\dfrac{1}{x} = ?$ $\quad x^2+\dfrac{1}{x^2} = ?$ $\quad x^3+\dfrac{1}{x^3} = ?$

Sol:-

$$x = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}}$$

$$\frac{1}{x} = \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$x+\frac{1}{x} = \frac{\sqrt{5}-\sqrt{2}}{\sqrt{5}+\sqrt{2}} + \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$$

$$= \frac{(\sqrt{5}-\sqrt{2})^2+(\sqrt{5}+\sqrt{2})^2}{(\sqrt{5}+\sqrt{2})(\sqrt{5}-\sqrt{2})}$$

$$= \frac{(\sqrt{5})^2+(\sqrt{2})^2-2(\sqrt{5})(\sqrt{2})+(\sqrt{5})^2+(\sqrt{2})^2+2(\sqrt{5})(\sqrt{2})}{(\sqrt{5})^2-(\sqrt{2})^2}$$

$$x+\frac{1}{x} = \frac{5+2-\cancel{2\sqrt{10}}+5+2+\cancel{2\sqrt{10}}}{}$$

$$\boxed{x+\frac{1}{x} = \frac{14}{3}}^{\,3}$$

ii) $x + \frac{1}{x} = \frac{14}{3}$

$$\left(x + \frac{1}{x}\right)^2 = \left(\frac{14}{3}\right)^2$$

$$x^2 + \frac{1}{x^2} + 2(\cancel{x})\left(\frac{1}{\cancel{x}}\right) = \frac{196}{9}$$

$$x^2 + \frac{1}{x^2} = \frac{196}{9} - \frac{2}{1}$$

$$x^2 + \frac{1}{x^2} = \frac{196 - 18}{9}$$

$$\boxed{x^2 + \frac{1}{x^2} = \frac{178}{9}}$$

$$x+\frac{1}{x}=\frac{14}{3} \qquad x^3+\frac{1}{x^3}=?$$

$$\left(x+\frac{1}{x}\right)^3=\left(\frac{14}{3}\right)^3$$

$$(x)^3+\left(\frac{1}{x}\right)^3+3(\cancel{x})\left(\frac{1}{\cancel{x}}\right)\left(x+\frac{1}{x}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+\cancel{3}\left(\frac{14}{\cancel{3}}\right)=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}+14=\frac{2744}{27}$$

$$x^3+\frac{1}{x^3}=\frac{2744}{27}-14$$

$$\Longrightarrow x^3+\frac{1}{x^3}=\frac{2744-378}{27}$$

$$\boxed{x^3+\frac{1}{x^3}=\frac{2366}{27}}$$

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6) $$\frac{\sqrt{3}-1}{\sqrt{3}+1}+\frac{\sqrt{3}+1}{\sqrt{3}-1}=a+b\sqrt{3}$$

$a=?$

$b=?$

sol:- $$\frac{(\sqrt{3}-1)^2+(\sqrt{3}+1)^2}{(\sqrt{3}+1)(\sqrt{3}-1)}=a+b\sqrt{3}$$

$$\frac{(\sqrt{3})^2+(1)^2-\cancel{2\sqrt{3}}+(\sqrt{3})^2+(1)^2+\cancel{2\sqrt{3}}}{(\sqrt{3})^2-(1)^2}=a+b\sqrt{3}$$

$$\frac{3+1+3+1}{3-1}=a+b\sqrt{3}$$

$$\frac{\cancel{8}\,4}{\cancel{2}}=a+b\sqrt{3}$$

$$4+0\sqrt{3}=a+b\sqrt{3}$$

comparing The coefficients

$\boxed{a=4}$ $\boxed{b=0}$